

DESIGN AND SIMULATION OF FLOATING POINT 32 INPUTS SPLIT-RADIX ALGORITHM

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ABSTRACT

Fast Fourier Transform plays a very important role in many engineering applications. Many different algorithms are proposed to improve the architecture of FFT among them Split-radix FFT is a particular FFT algorithm that aims to compute FFT with the least number of multiplications. The split-radix FFT mixes radix-2 and radix-4 decompositions, yielding an algorithm with about one-third fewer multiplies than the radix-2 FFT. The split-radix FFT has lower complexity than the radix-4 or any higher-radix power-of-two FFT. This paper proposes split-radix FFT for both real and floating point numbers. In this paper we use verilog for code implementation. The FPGA synthesis and logic simulation is displayed in Xilinx design suit13.2. The results shows the improvement in the speed

KEYWORDS: Split, Radix, Fast Fourier Transform, Butter Fly, Verilog, Floating Point, FPGA

INTRODUCTION

Fast Fourier Transform (FFT) has become ubiquitous in many engineering applications. High-speed FFT operations are necessary in the field of digital signal processing, digital image processing applications and also in several communication systems. Fast Fourier Transform (FFT) is an efficient algorithm to compute the discrete Fourier transform (DFT) and it's inverse. It makes the use of the symmetry properties of twiddle factor to effectively reduce the DFT computation time. It is based on fundamental principle of decomposing the computation of DFT of a sequence of length N into successively smaller discrete Fourier transforms

The N-point DFT of sequence $x(n)$ is given by

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{kn} \quad 0 \leq K \leq N-1$$

Where $W_N = e^{-j2\pi/N}$ is known as twiddle factor

The efficiency of the FFT algorithm lies in its reduced number of arithmetic operations. DFT has the order of $O(N^2)$ arithmetic operations whereas FFT has the order of $O(N \log N)$ arithmetic operations. The **Cooley - Tukey** algorithm was proposed in 1965 which is used to divide the transform into two pieces of size $N/2$ at each step, and is therefore limited to power of two sizes. This algorithm is usually called radix-2 algorithm

Radix-2 Algorithm

In this radix-2 algorithm N-point sequence decimates into two sequences of length $N/2$, where one sequence consists of the even-indexed values of $x(n)$ and other odd indexed values of $x(n)$

DIT radix-2 representation is

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi nk}{N}}$$

$$X(K) = \sum_{n=0}^{N/2-1} x(2n) e^{-j\frac{2\pi(2n)k}{N}} + \sum_{n=0}^{N/2-1} x(2n+1) e^{-j\frac{2\pi(2n+1)k}{N}}$$

$$X(K) = \text{DFT}[\text{even numbered indices}] + W_N^k \text{DFT}[\text{odd numbered indices}]$$

DIF radix -2 representation is

$$X(2K) = \sum_{n=0}^{N/2-1} [x(n) + x(n + \frac{N}{2})] W_{N/2}^{nk} \quad K=0,1,\dots,N/2-1$$

$$X(2K+1) = \sum_{n=0}^{N/2-1} [x(n) - x(n + \frac{N}{2})] W_N^n W_{N/2}^{nk} \quad k=0,1,\dots,N/2-1$$

Radix-4 Algorithm

When the number of samples N in the input sequence is a power of 4, it is more computationally efficient to employ a radix-4 FFT algorithm. The radix-4 decimation-in-frequency (DIF) FFT are as follows. we subdivide $x(k)$ into four $N/4$ -point sub sequences $x(4k)$, $x(4k+1)$, $x(4k+2)$ and $x(4k+3)$ of each length 0, 1, ..., $N/4-1$.

So, the radix-4 DIF DFT are obtained as follows

$$X(4K) = \sum_{n=0}^{N/4-1} [x(n) + x(n + \frac{N}{4}) + x(n + \frac{N}{2}) + x(n + \frac{3N}{4})] W_N^0 W_{N/4}^{nk}$$

$$X(4K+1) = \sum_{n=0}^{N/4-1} [x(n) - jx(n + \frac{N}{4}) - x(n + \frac{N}{2}) + jx(n + \frac{3N}{4})] W_N^n W_{N/4}^{nk}$$

$$X(4K+2) = \sum_{n=0}^{N/4-1} [x(n) - x(n + \frac{N}{4}) + x(n + \frac{N}{2}) - x(n + \frac{3N}{4})] W_N^{2n} W_{N/4}^{nk}$$

$$X(4K+3) = \sum_{n=0}^{N/4-1} [x(n) + jx(n + \frac{N}{4}) - x(n + \frac{N}{2}) - jx(n + \frac{3N}{4})] W_N^{3n} W_{N/4}^{nk}$$

Split-Radix FFT

The split-radix algorithm, first clearly described and named by **Duhamel and Hollman** in 1984, required fewer total multiply and add operations than any previous power-of-two algorithm. Split-radix FFT is one of the FFT algorithms that use combination of different radix FFT. Split-radix FFT algorithm combines simplicity of radix-2 FFT with less computational complexity radix-4 FFT.

The advantage of split-radix FFT is that it has considerably fewer number of arithmetic computations compared to that of radix-4 and radix-2 FFT. Split-radix also has several other advantages such as regular structure, no reordering of internal signals except for outputs, etc.

The split-radix algorithm improves the arithmetic complexity of the Cooley-Tukey algorithm by further decomposing the odd parts into odd odd and odd-even parts, while the even parts are left alone because they have no multiplicative factor. Even-numbered samples of the N -point DFT can be calculated by radix-2

$$X(2K) = \sum_{n=0}^{N/2-1} [x(n) + x(n + \frac{N}{2})] W_{N/2}^{nk} \quad K=0,1,\dots,N/2-1$$

The odd-numbered samples $x(2K+1)$ requires an additional multiplication of W_N^n . To implement this, radix-4 algorithm is used for its lesser computational complexity. Using radix-4 algorithm for the odd-numbered samples of the N -point DFT, the following $N/4$ -point DFTs are

$$X(4K+1) = \sum_{n=0}^{N/4-1} [x(n) - jx(n + \frac{N}{4}) - x(n + \frac{N}{2}) + jx(n + \frac{3N}{4})] W_N^n W_{N/4}^{nk}$$

Where $K=0,1,\dots,N/4-1$ and

$$X(4K+3) = \sum_{n=0}^{N/4-1} [x(n) + jx\left(n + \frac{N}{4}\right) - x\left(n + \frac{N}{2}\right) - jx\left(n + \frac{3N}{4}\right)] W_N^{3n} W_N^{nk}$$

Where $K=0,\dots,N/4-1$

Hence, the N -point DFT now has been decomposed into one $N/2$ -point DFT without phase factor and another two $N/4$ -point DFTs with phase factor. Figure 1 shows the split-radix butterfly unit.

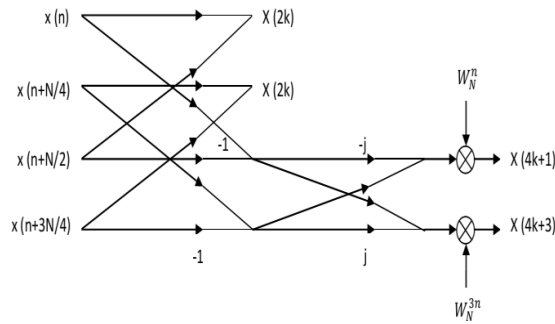


Figure 1: Split-Radix Butterfly Unit

In this paper we implemented the split-radix fft for 32 input. The below figure 2 represents the architecture for 32input split-radix .In every block represents split-radix unit

FPGA

In this paper the proposed 32-input split radix has been simulated and synthesised in using the Xilinx design suite 13.2. In this proposed paper the device family is vertex-7.

About vertex-7

7 series FPGAs are offered exclusively in thermally efficient flip-chip BGA packages. These 0.5 mm, 0.8 mm, and 1.00 mm flip-chip packages range in pin-count from the smaller 10 x 10 mm CPG236 to the 45 x 45 mm FF1930. The suite of packages is used to address the various power requirements of the 7 series devices. All 7 series devices are implemented in the 28 nm process technology.

Target Device : xc7k160t-3-fbg676

Packaging details: FF (G) 676

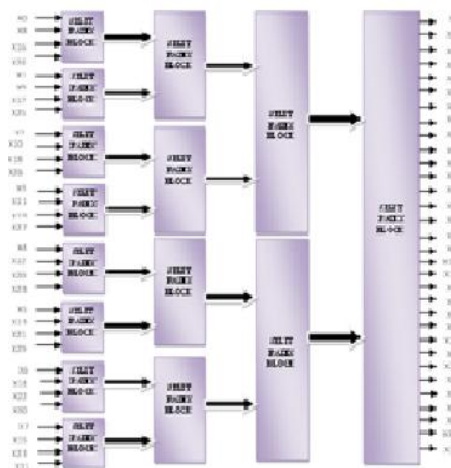


Figure 2: 32 Input Split-Radix Implementation Diagram

SIMULATION RESULTS

The below diagram figure 3 represent RTL view of internal split radix block

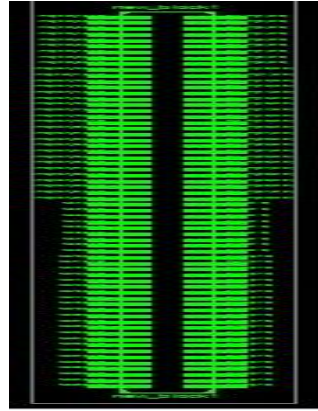


Figure 3: RTL View of Internal Split- Radix FFT

The below figures 5,6,7,8 represents the simulation results of internal split-radix blocks. The inputs given in the form both real and floating point numbers. Inputs are from real1 to real32 & imaginary1 to imaginary32. The out puts are form real_1 to real_32& imaginary _1 to imaginary_32

Name	Value	1999,995 ps	1999,996 ps	1999,997 ps	1999,998 ps	1999,999 ps
real_1[31:0]	01000011010		01000011010	00000000000000000000		
real_2[31:0]	11000001000		11000001000	110011010101010101		
real_3[31:0]	11111111000		11111111000	10010110010000001		
real_4[31:0]	11111111000		11111111000	10110110010000001		
real_5[31:0]	01000011000		01000011000	11111000000000000000		
real_6[31:0]	11111111011		11111111011	100000111011001111100		
real_7[31:0]	11111111100		11111111100	10111010111000001		
real_8[31:0]	11111111100		11111111100	111110111000001		
real_9[31:0]	01000001101		01000001101	1001100110011001100		
real_10[31:0]	11111111101		11111111101	11110011100110011001		
real_11[31:0]	11111111101		11111111101	1100000100011111		
real_12[31:0]	01111111101		01111111101	10000010001111		
real_13[31:0]	11110001010		11110001010	1001100110011001		
real_14[31:0]	01000000011		01000000011	1001100110011001		
real_15[31:0]	11111111001		11111111001	1110000000000001		
real_16[31:0]	11111111001		11111111001	1110000000000001		

Figure 4: Simulation Results of First Split-Radix FFT Block

Name	Value	1999,995 ps	1999,996 ps	1999,997 ps	1999,998 ps	1999,999 ps
real_17[31:0]	01111111011		01111111011	1100001100100100		
real_18[31:0]	11000001000		11000001000	1101101101110		
real_19[31:0]	10000000110		10000000110	10101010101010		
real_20[31:0]	01111111000		01111111000	11011110000		
real_21[31:0]	11111111111		11111111111	1101011101001		
real_22[31:0]	00000000001		00000000001	10101010101010		
real_23[31:0]	01111111001		01111111001	110110011100		
real_24[31:0]	00000000110		00000000110	110100000111010		
real_25[31:0]	00111111111		00111111111	1100001010100011		
real_26[31:0]	11111111101		11111111101	1000000101000110		
real_27[31:0]	10111111111		10111111111	1100001010100100		
real_28[31:0]	00111111111		00111111111	1100001010100100		
real_29[31:0]	01000000111		01000000111	1100001100010		
real_30[31:0]	01000000110		01000000110	1011011110100		
real_31[31:0]	11111111100		11111111100	1000010101010		
real_32[31:0]	11111111100		11111111100	1000010101010		

Figure 5: Simulation Results of Second Split-Radix FFT Block

Name	Value	999,995 ps	999,996 ps	999,997 ps	999,998 ps	999,999 ps
real_1[31:0]	01000011010		01000011010	010000000000000000000000		
real_2[31:0]	11000001000		11000001000	1111011010101010101010101		
real_3[31:0]	11111111000		11111111000	1001001001001000000001		
real_4[31:0]	11111111000		11111111000	100100100101010100000001		
real_5[31:0]	01000011000		01000011000	111100000000000000000000		
real_6[31:0]	11111111011		11111111011	1000001110110011111100		
real_7[31:0]	11111111000		11111111000	1010101101011100000001		
real_8[31:0]	11111111000		11111111000	101011100110000001		
real_9[31:0]	01000001101		01000001101	1001100110011001100		
real_10[31:0]	11111111010		11111111010	100100011100110101001		
real_11[31:0]	11111111010		11111111010	11100000100011111		
real_12[31:0]	01111111010		01111111010	110000010001111		
real_13[31:0]	11110010101		11110010101	1001100110011010001		
real_14[31:0]	01000000011		01000000011	100110011000110100		
real_15[31:0]	11111111001		11111111001	1110000000000001		
real_16[31:0]	11111111001		11111111001	1110000000000001		

Figure 6: Simulation Result of Third Split-Radix FFT Block

Name	Value	999,995 ps	999,996 ps	999,997 ps	999,998 ps	999,999 ps
imaginary_16[31:0]	01110011100		01110011100	11011011001010001		
imaginary_17[31:0]	11111111011		11111111011	1010000101000000011		
imaginary_18[31:0]	10000000000		10000000000	1000011101000010101		
imaginary_19[31:0]	01111111011		01111111011	1000010100000100		
imaginary_20[31:0]	01111111011		01111111011	1000010100000100		
imaginary_21[31:0]	01111111000		01111111000	111000010101		
imaginary_22[31:0]	01111111111		01111111111	101000000101		
imaginary_23[31:0]	11111111110		11111111110	1001010010101		
imaginary_24[31:0]	11111111111		11111111111	1111111101		
imaginary_25[31:0]	01111111111		01111111111	100100010111001		
imaginary_26[31:0]	10000000001		10000000001	100100100000		
imaginary_27[31:0]	10000000010		10000000010	111111111		
imaginary_28[31:0]	11111111000		11111111000	111100110100110		
imaginary_29[31:0]	11111111100		11111111100	1000010001		
imaginary_30[31:0]	11111111100		11111111100	1000010101		

Figure 7: Simulation Result of Fourth Split -Radix FFT Block

SYNTHESIS REPORT

The final synthesis report for 32-point SRFFT for both real and floating point inputs is given below the device utilization values & time delay for proposed paper

Table 1: Device Utilization Summary of Vertex-7 for 32-Point SRFFT

Logic Utilization	Used	Available	Utilization
Number of slice LUTs	130048	101400	128%
Number of fully used LUT-FF pairs	3840	60405	6%
Number of bonded IOBs	4096	400	1024%
Number of DSP48E1s	197	600	31%

Table 2: Time Delay of 32-Point SRFFT

Time delay	17.2517ns
The total REAL & CPU time toXs: complion:	TotalREAL time to Xst completion: 310.00 secs TotalCPU time to Xst completion: 310.2175 secs Totalmemory usage is 11,91,447kilobytes

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